Conditional Value at Risk and Extreme Values: 
Risk Analysis of the Emerging Stock Markets 
from Brazil and Mexico 

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Abstract

Stock markets, particularly those from the developing countries, are characterized by high volatility which conventional models fail to capture fully, potentially leading to high losses. Value at Risk (VaR) models signified an important step to estimate losses of financial assets and portfolios. However, the stylized fact that financial returns exhibit fat tails, implies that conventional VaR models (parametrics and non-parametrics models) show important limitations because they fail to take into account the right statistical distributions to capture the frequency and severity of extreme values; the normal distribution is insufficient for this purpose. Extreme Value Theory (EVT) overcomes this limitation because it provides a framework to formally study the extreme behavior of stock markets returns and quantifies the possible losses experienced during financial instabilities and turbulences without making any assumptions about the underlying distribution of returns. This study uses the c-quantile of a fat tailed distribution to Value at Risk analysis; the parameters thus obtained improve the accuracy of measuring financial risk and reduces model risk. This work also extends the results of extreme value theory to show the impact of the frequency and size of extreme returns on values of Conditional Value at Risk (CVaR). A generalized extreme value distribution (GEVD) is used for the cases of the two largest stock markets from Latin America, Brazil and Mexico, and a CVaR model is applied to determine risk exposure by investing in those markets. Daily index data for the period January 2 1970 to December 31, 2004 is used. The empirical evidence shows a high potential of GEVD to explain adequately the extreme behavior from the stock markets from Brazil and Mexico. Similarly, analyzing tail indexes estimates for each market, weak evidence of asymmetry between left and right tails is found; the distribution’s right tail is slightly heavier than the left tail, which is different to the case of mature markets. Also, the return distribution from the Mexican Stock Exchange presents heavier tails and a higher propensity to experience losses than the distribution from the Brazilian exchange. Finally, in general, Extreme Value Theory is a more conservative methodology to estimate VaR and CVaR than the traditional VaR models.

Key Words: Extreme Values, Value at Risk, Risk Management, Emerging Capital Markets, Mexico, Brazil.

JEL Classification: C-16, C-51; G-15; G-32

1. Introduction

Risk management has become one of the most important areas in contemporary finance. Financial and economic globalization have open the door to many investment opportunities in both mature and emerging markets and hence greater diversification schemes aiming at optimizing portfolio returns. However, a very complex environment influences portfolio holdings: trends in domestic and international variables determine asset prices. Potential losses
might result from adverse trends in interest rates, exchange rates, inflation rates, and high volatility of prices of goods and services, along with risk factors derived from local and international social and political variables.

In response to these challenges risk management has developed important tools to measure and control risk to prevent potential losses. In practice, Value at Risk (VaR) has become one of the most important tools to manage risk. Parametric and non-parametric (simulation) VaR models have been developed in the financial literature. However, these models are limited to cope with large instabilities like those recently facing international financial markets. Indeed large portfolio losses have taken place derived from the financial crises affecting the world markets in recent decades. The presence of atypical and unexpected movements limits the efficiency of conventional VaR models due to excess kurtosis present in the distributions of returns (Duffie and Pan, 1997). Similarly, GARCH models with normal innovations or t-student distributions introduced by Jorion (1988), and Yang and Brorsen (1995) have proved insufficient to capture different degrees of leptokurtosis, as well as asymmetries of risk factors. Although non-parametric VaR models yield more reliable results than the conventional delta-normal technique they are limited in adjusting for extreme returns, present in a returns distribution, which is very important to measure risk adequately in order to insure survival of financial institutions and the stability of financial systems (Hendricks, 1996; Jackson, Maude and Perraudin, 1997; Vlaar, 2000). In short, VaR techniques fail to estimate extraordinary losses that might arise at times of financial turbulences. This shortcoming has been overcome with Conditional Value at Risk model (CVaR), introduced by Artzner et al (1999), which measures the expected shortfall. These measures have been presented in the financial literature as coherent risk measures because they share the same proprieties when applied to continuous distributions. CVaR estimates the tail risk in a conservative and efficient manner by incorporating both the frequency as well as the size of extreme events. Nevertheless, assuming normality, for high confidence levels CVaR underestimates risk because it fails to incorporate all the information from the tails of a returns distribution. In response to these limitations Extreme Value Theory (EVT) sets forth sound techniques to understand and model the statistical behavior of extreme values. Longin (1994, 2000) pioneered the application of EVT to VaR analysis and other researchers have followed. The generalized extreme values distribution (GEVD), first developed by Jenkinson (1969) and Longin (2001), has been applied to estimate daily VaR for the S&P stock index. In the case of emerging capital markets the application of EVT has remained very limited. This work is a contribution in this area. Its purpose is two-folded.
First, it examines the size and frequency of extreme returns for the case of the stock market indexes from Brazil and Mexico for the period 1970-2004 applying the generalized extreme value distribution. Second, it applies TVE to CVaR analysis for short and long positions and compares the results with conventional VaR estimates. The paper is organized as follows. Following this introduction, Section 2 reviews the literature. Section 3 present the models used for the empirical research. Section 4 reports the empirical findings. The conclusions are in Section 5

2. Literature Review

A large body of literature evidences the presence of excess kurtosis in financial series, the heavy or fat tails effect, as well as of different degrees of asymmetry; Mandelbrot (1963) and Fama (1965) pioneered this research. Several alternative distributions have been proposed to solve the problem of fat tails. A fat tails distribution proposed to model extreme movements is the mixture of normal distributions suggested by Boness et al (1974). The application of this model for risk analysis using VaR was described first by Zangari (1996); later Ventakaraman (1997) improved this model by estimating its parameters using quasi-bayesian maximum likelihood estimation. More recently, a clear fat tails model applied to VaR analysis was introduced by Hull and White (1998). Similarly, Huisman, Koedijk and Pownall (1998) and Heikkinen and Kanto (2002) proposed the student’s t-distribution to capture excess risk combining the finite variance with the fat tails distribution of returns. Andrew and Kanto (2005) developed a closed model to determine impacts from leptokurtosis in CVaR estimation. These alternatives to the normal distribution model solve fairly the phenomena of heavy tails and yield better risk estimates; however they present several disadvantages, mainly the lack of closed expressions; besides their models neglect adjusting in the distribution the asymmetry levels observed in financial returns. Responding to inconsistencies from previous models, to capture the magnitude and probability of extreme values the theory of extreme events provides a set of sound tools to assess and model risk, present in empirical distributions. Although EVT has a long standing in the scientific world, solving successfully problems related to hydrology, weather changes, and insurance, it is only recently that has began to be applied in financial economics to model the impact of tail events on portfolio holdings. Empirical studies applying EVT to examine the behavior of returns to changes in exchange rates include Danielsson and de Vries (1997), Loretan and Phillips (1994), Hols and de Vries (1991) and Koedijk, Shanfgans y de Vries (1990). Interest in EVT for risk analysis and management to measure the behavior of
financial markets has grown considerably during the last decade, particularly for the case of mature markets. Longin (1996, 2000) pioneered the application of EVT in VaR analysis, using the generalized extreme values distribution, estimating VaR for daily returns of the S&P stock market index. A more recent study, based in a hybrid model, estimating the tail index and quantifying VaR for random portfolios is that from Danielsson and de Vries (2000). McNeil (1999) and McNeil and Frey (2000) used daily data to illustrate the potential of the generalized Pareto distribution (GPD) to measure tail risk related to the empirical distributions, for the case of DAX and S&P stock indexes. The behavior of interest rates extreme values for the case of Treasury Bills and the LIBOR rate have been studies by Neftci (2000), Bali y Neftci (2001) y Krehbiel y Adkins (2005). In the case of emerging markets important works are those by Jondeau and Rockinger (2003), Susmel (2001), Gencay and Selcuk (2004), Fernández (2003), da Silva and Mendes (2003) and Ho et al. (2000). Jondeau and Rockinger, and Susmel use GPD to compare the asymptotic behavior from mature and emerging stock markets; Gencay and Selcuk estimate VaR using EVT for several emerging markets; Fernandez presents empirical evidence for the case of the Chilean stock market; finally, da Silva y Mendes (2003), and Ho et al (2000) based in the construction of block maxima examined potential losses at the Asian stock markets. Nevertheless, the literature related to extreme returns associated with booms and crashes for the case of the emerging Latin American stock markets is scarce, even though financial authorities and investors have a growing need to assess risks related to their institutions and markets.

3. Data and Methodology

3.1 Data

The dynamics of extreme movements in high frequency financial series, which allows estimating a distribution’s tail risk, are analyzed in this work using daily data from Brazilian and Mexican emerging stock markets. Daily data was gathered for the period January 2, 1970 through December 31, 2004. A total of 8772 observations for the Mexican Stock Market Index (Indice de Precios y Cotizaciones, IPC) and the Sao Paolo bourse (BOVESPA), were from the Reuters Data Bank, complemented with local information from the Central Banks from these countries. Thirty-five years of daily data constitute a fine sample to carry out a thorough analysis concerning extreme movements and risk in Brazil and Mexico. The series include, for instance, financial crises such as instabilities due to the debt crisis from the 1980's, the world wide 1987 stock markets crisis, the Mexican 1994-1995 crisis, the Asian countries crisis from 1997, the Russian crisis from 1998, the Brazilian crisis from 1999, and the Argentinean crisis from 2001.
3.2 Methodology

Risk analysis and management has evolved to the point that investors are now concerned with the impact of low probability events in their portfolios. Particularly, massive losses might be caused by extreme movements. In this context conventional Value at Risk (VaR) estimates are inappropriate, limited to model extreme movements captured on the tails of the returns distributions. Extreme Value Theory overcomes such shortcomings. This theory studies the asymptotic behavior from extreme observations in order to develop rational procedures that serve as a basis to estimate with greater precision extreme quantiles of a distribution, without a need to know the full distribution.

3.3 Univariate Extreme Value Theory

Let a set of random variables $R_1, R_2, \ldots, R_n$ represent the daily observations on the returns of a stock market index, and $M_n = \text{Max}\{R_1, R_2, \ldots, R_n\}$ the maximum return that can attain a financial asset during $n$ days of trade. If returns are independent and identically distributed and belong to an unknown distribution $F_R$, then the distribution for maximum, $M_n$ \(^1\) can be expressed as follows:

$$P\{M_n \leq r\} = P\{R_1 \leq r, R_2 \leq r, \ldots, R_n \leq r\} = \prod_{j=1}^{n} P\{R_j \leq r\} = F_R^n(r) = F_{M_n}(r). \quad (1)$$

This expression stresses the viability of obtaining the distribution function for the maximum for a finite simple when the distribution $F_R$ is known. However, empirically the use of equation (1) is limited. Based on empirical existing evidence it can be affirmed that not only the returns distribution is unknown, but also the distribution for the maximum (Longin and Solnik, 2001). Thus, the distribution $F_R^n$ is degenerated as $n \to \infty$. To solve this problem the random variable $M_n$ must be transformed so that the asymptotic distribution of the new random variable becomes a non-degenerated distribution. This can be obtained with a simple normalization operation, finding out two sequences of positive numbers $\{\beta_n\}$ and $\{\alpha_n\}$, and $\alpha_n > 0$ that

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\(^1\) Risk management also requires models to describe the behavior of minimum returns due to its importance for VaR estimation. However the remaining of this section only present the theoretical results for the maximum because results for the minimum can be deduced from the maximum by only changing the sign, that is, $\text{Max}\{R_1, R_2, \ldots, R_n\} = -\text{Min}\{-R_1, -R_2, \ldots, -R_n\}$. 

stabilize or adjust the location and scale parameters for \( M_n^* = \frac{M_n - \beta_n}{\alpha_n} \), so that converges in distribution to some non-degenerated random variable as \( n \to \infty \). This is one of the fundamental principles from EVT established by Fisher y Tippett (1928)\(^2\) and finally demonstrated by Gnedenko (1943).

Suppose that \( R_1, R_2, K, R_n \) are random variables independent and identically distributed with a function of distribution \( F_R(r) \), and let \( R_{n1}, R_{n2}, K, R_{nm} \) be its order statistics. If there are the normalization successions \( \{\beta_n\} \) and \( \{\alpha_n\} \), with \( \alpha_n > 0 \), so that

\[
P\left(\frac{M_n - \beta_n}{\alpha_n} \leq r\right) = F^n(\alpha_n r + \beta_n) \xrightarrow{d} H(r),
\]

where \( H \) is a non-degenerated distribution, then \( H \) belongs to one of the following families.\(^3\)

**Type I:**

\[
H(r) = \exp\{-\exp(-r)\}, \quad -\infty < r < \infty;
\]

**Type II:**

\[
H(r) = \begin{cases}
\exp\{-\left(r^{-\tau}\right)\}, & r > 0, \quad \tau > 0 \\
0, & r \leq 0
\end{cases}
\]

**Type III:**

\[
H(r) = \begin{cases}
\exp\{-\left(r^\tau\right)\}, & r \leq 0, \quad \tau < 0 \\
1, & r > 0
\end{cases}
\]

Thus, the distribution \( F_R \) is in the maximum domain of attraction of the limiting distribution \( H \). This family of distributions are known as the distributions of standard extreme value. Type I is known as the Gumbel (1958) distribution or else as distribution of light tails,

\(^2\) This result from the Central Limit Theorem which studies the asymptotic behavior of the maximum of a set of random variables in lieu of the asymptotic behavior of the partial sum.

\(^3\) The notation "\( \xrightarrow{d} \)" means convergence in distribution. See: Mittnik y Rachev (1993).
because all its moments are finite and the behavior of the tails of the distribution decays exponentially. Type II is defined as the Fréchet (1928) distribution or distribution of heavy tails. Gnedenko (1943) evidenced that if the tails from $F_{\xi}(r)$ decay slowly, then is at the maximum domain of attraction of the Fréchet distribution. Finally, Type III is known as the Weibull (1951) distribution or distribution of short or bounded tails. This distribution is characterized by the fact that has a finite right endpoint. The form parameter $\tau$ plays a key role in the identification on the shape of the limiting distribution, i.e. it represents the weight of the tail of the distribution $H(r)$.

Nevertheless, one of the main problems in the analysis of extreme values is the determination of the form parameter $\tau$. This problem can be easily solved reformulating the models of the Fisher and Tippett (1928) theorem reparametrizing $\xi = -1/\tau$ known as the tail index. For estimation purposes it is very useful alternative representation for the three types of standard extreme value distributions, which can be expressed as follows:

$$H_{\xi}(r) = \begin{cases} 
\exp\left[-(1-\xi r)^{\frac{1}{\xi}}\right] & \text{si } \xi \neq 0 \\
\exp[-\exp(r)] & \text{si } \xi = 0,
\end{cases}$$

(2)

for $r < 1/\xi$ if $\xi < 0$ and for $r > 1/\xi$ if $\xi > 0$. This family of distribution functions depending of the tail index $\xi$, commonly known as the Generalized Extreme Value Distribution (GEVD) was first introduced independently by Mises (1936) and Jenkinson (1955), and reparametrized to a more general context by Maritz and Munro (1967).

In this way the value of the tail index plays a key role for discriminating standard extreme value distributions. If $\xi_n < 0$ the distribution function $F_{\xi_n}(r)$ is at the maximum domain of attraction of the Fréchet distribution, which is generally valid to model financial series. There are special cases of distributions whose tails decay in a polynomial manner, which include the $\alpha$-stable, Cauchy, t-student distributions and the mixture of normal distributions, among the most important. When $\xi_n > 0$ it is said that the distribution function $F_{\xi_n}(r)$ belongs to the maximum domain of attraction of the Weibull distribution, which is not very efficient to explain the behavior of financial return series. Some examples of this type of distributions are the uniform and beta distributions. Finally, if $\xi_n = 0$, the distribution function $F_{\xi_n}(r)$ is at the maximum domain of attraction of the Gumbel distribution, which includes the normal,
exponential, gamma distributions and lognormal distribution, which has much heavier tails. Similarly if the location and scale parameters $\beta_n$ and $\alpha_n > 0$ are considered, it is possible to extend the family of distributions such that $H_{\xi}(r) = H_{\alpha_n, \beta_n, \theta_n}(r^{\frac{\beta_n}{\alpha_n}})$, an important result for the analysis of extreme values supported by the Fisher and Tippet Theorem (1928), which states that if $F_R \in MDA(H)$, then $H$ is of the same type of $H_{\xi}$ for some $\xi$.

The generalized extreme value distribution depends on the behavior of a family of parameters. First is defined the location parameter $\beta_n$ which indicates where are on the average located the extreme values. Second is the scale parameter $\alpha_n$ which determines the dispersion of the extreme observations. Lastly, is the tail index $\xi_n$, which describes the behavior of the tail of the limiting distribution, generally denoted by $\tau = -\frac{1}{\xi_n}$.

### 3.4 Maximum Likelihood Method for Extreme Returns

One of the crucial challenges in measuring the tail risk, presented in a number of empirical financial applications refers to the values of estimated parameters of the generalized extreme value distribution, particularly the tail index. To estimate the three relevant parameters there are several standard statistical techniques such as probability weighted moment estimation, maximum likelihood estimation, and regression analysis. In this paper the estimation of the parameters of location $\beta_n$, scale $\alpha_n$ and tail index $\xi_n$ is carried out using the maximum likelihood method. Longin (1996, 2000), advices to divide the total sample in several subsamples. Assume a sample of returns $r_1, r_2, K, r_N$, is observed which is a realization of random variables independently and identically distributed. Dividing the total sample by $m$ which represents the number of subsamples with $n$ observations each one. In other words, if $N = mn$, the $i$th subsample of the returns series is defined by

$\{r_1, r_2, K, r^n, r^{n+1}, r^{n+2}, K, r^{2n}, \ldots, r_{(m-1)n+1}, r_{(m-1)n+2}, K, r_{mn}\}$. Supposing that $r_{n,i}$ represents the maximum value of the $i$th subsample, then a set of maximum subsamples can be obtained from the sample of size $m$. Assuming independence, the log-likelihood function for the collection of maximum subsamples can be expressed as follows,
where $h_{\xi, \beta, \alpha}(r_{n,j})$ is defined as the probability density function of the generalized extreme value distribution. For $\xi_n > -0.5$, the family of the generalized extreme value distribution satisfies all conditions of regularity as shown by Smith (1985). That is, the maximum likelihood estimators are consistent, efficient and asymptotic normal, even if the data are not independent and identically distributed. Similarly, this method helps in calculating standard errors, construct confidence intervals, as well as carrying out other inferences related to the GEV parameters $(\xi_n, \beta_n, \alpha_n)$ follow immediately from the approximate normality maximum likelihood estimators. Nevertheless the statistical procedure of block maxima presents a trade-off problem between bias and variance. A sufficiently large block does not only lead to a more precise model estimation, but also it yields estimators with low bias and high variance because extreme values are rare by definition. With a small block the number of observations in the sample increases which reduces the variance, there remains the possibility that the asymptotic properties of the model estimation is violated.

### 3.5 Tests of Goodness of Fit

Once the parameters of a family of distributions have been estimated it is important to quantify the uncertainty derived from the estimations made. Test of goodness-of fit on the probability distributions must be carried out, i.e. a statistical validation of how well the model describes or explains the behavior of the available data.

Several formal tests have been presented in the literature, the large majority is based in comparing the estimated distribution with the observed distribution. In this case, Sherman (1957) proposed a test of goodness of fit based in a series of ordered data denoted by $(r_{n,j})_{j=1,K,m}$. Calculation of the statistic is determined in the following way,

$$
\Omega_m = \frac{1}{2} \sum_{i=0}^{m} H_{\xi, \beta, \alpha} (r_{n,i+1}) - H_{\xi, \beta, \alpha} (r_{n,i}) - \frac{1}{m+1},
$$

(5)
with \( H_{\xi, \beta, \alpha} (r_n, 0) = 0 \) and \( H_{\xi, \beta, \alpha} (r_{n+1}) = 1 \). The statistic \( \Omega_m \) is distributed asymptotically like a normal variable with mean \( \left( m / (m + 1) \right)^{m+1} \) and variance \( \left( 2e - 5 \right) / me^2 \). A low \( \Omega_m \) statistic indicates that the discrepancy between the estimated distribution and the observed distribution is small, in other words, extreme value theory describes well the behavior of extreme returns.

Another common statistic that aids testing whether the maximum likelihood estimates are statistically significant is the test of the likelihood ratio defined by

\[
\lambda = 2 \left( l(\hat{\theta}) - l(\hat{\theta}^*) \right),
\]

where \( l(\hat{\theta}) \) is the maximum value of the log-likelihood function for the generalized extreme value function, or non restricted model, and \( l(\hat{\theta}^*) \) is the maximum value of the log-likelihood function for the Gumbel distribution, or restricted model. To test the validity of the non restricted model represented by GEVD suffices to verify if \( \lambda > C_\alpha \), where \( C_\alpha \) is the \((1 - \alpha)\% \) percentile of the \( \chi^2 \) distribution with one degree of freedom because it only has one restriction.

### 3.6 VaR y CVaR and Models Based on GEVD

VaR measures maximum probable losses for a portfolio holdings, for a determined time horizon, given a confidence level. VaR is determined by the \( c \)–quantile of the returns distribution \( F \) with negative sign:

\[
\text{VaR}_c = -F^{-1}(c).
\]

Parametric and non parametric (simulation) VaR models have been developed in the financial literature. VaR measures, or \( c \)–quantile of the normal distribution of returns are a linear function of the variance and its implementation is an easy task. But risk estimations are incorrect, for they do not take into account extreme values captured at the tails of the return distributions. On the contrary using \( c \)–quantile from a heavy-tailed distribution yield better information about financial risks and therefore reduce modeling risks.

Assuming that extreme observations follow a generalized extreme value distribution, with a distribution function \( H_{\xi, \beta, \alpha} \), then VaR as a risk measure is represented by the quantile from the GEVD, which is obtained inverting such distribution and substituting the estimates of
maximum likelihood from the family of parameters \((\xi_n, \beta_n, \alpha_n)\). Thus, value at risk for maximum
returns corresponding to a short position can be expressed as

\[
\text{VaR}_c(X) = \hat{\beta}_n + \frac{\hat{\alpha}_n}{\xi_n} \left[ -n(-\ln c)^\frac{1}{\xi} \right].
\]  

(7)

Here, \(\hat{\alpha}_n\), \(\hat{\beta}_n\) and \(\xi_n\) represent estimators of maximum likelihood for the series of
maximum returns; \(c\) is the probability that the maximum return does not exceed the VaR level;
and \(n\) represents the size of the blocks from which are obtained maximum observed returns
during a determined number of days of operations. Choosing this parameter plays an important
role in VaR estimation based on classic EVT. For a long position following similar procedures
than for a short position can be expressed as follows.

\[
\text{VaR}_c(X) = -\hat{\alpha}_n + \frac{\hat{\beta}_n}{\xi_n} \left[ 1 - n(-\ln c)^\frac{1}{\xi} \right],
\]  

(8)

where \(\hat{\alpha}_n\), \(\hat{\beta}_n\) and \(\xi_n\) represent maximum likelihood estimates; \(c\) is the probability the minimum
return exceeds the VaR level; and \(n\) represent the size of the subsamples from which are
obtained the minimum observed returns for a determined number of days of operations.

A more refined alternative that measures excess losses above VaR has been developed
by Artzner et al (1997; 1999); Rockafellar and Uryasev (2000) named this model as Conditional
Value at Risk (CVaR). This model has been proposed because conventional VaR does not take
into account the statistical properties from extreme losses and additionally does not fulfill the
subadditivity principle, i.e. conventional VaR does not satisfy the properties of coherent risk
measures. Taking into account these properties, CVaR can be defined as

\[
\text{CVaR}_c(X) = -\text{E}\left[ X | X \leq \text{VaR}_c(X) \right]
\]  

(9)

In the context of EVT this risk measure can be estimated as a simulation of a function of
VaR for a specific returns distribution. This means that CVaR also depends of the returns
distribution \(F\) and from the probability level \(c\). The asymptotic relationship between these two
risk measures can be obtained analytically; taking advantage of the maximum domain of
attraction it can be constructed a heavy-tailed Pareto distribution, with location parameter equal
to zero, parameter of scale equal to one, and a characteristic exponent greater than one.
\((\alpha > 1)\); that is, \(F_x(x) = 1 - kx^{-\alpha}\), \(x \geq 0\) where \(k\) is a function that changes slowly and \(\alpha\) is a positive parameter known as the tail index of the distribution \(F\).

Considering the inequality \(x > u\) from the tail of distribution \(F\) it can be obtained the CVaR for a heavy-tailed Pareto distribution located at the maximum domain of attraction of the generalized extreme value distribution equal to

\[
\text{CVaR}_\alpha(x) = \left(\frac{\alpha}{\alpha - 1}\right) \text{VAR}_\alpha(x)
\]

(10)

Thus, the CVaR is always greater that VaR when a distribution of heavy tails is assumed, for the difference between both measures does not converge to zero when the values from the tails of the returns distribution are taken into account. This result depends in greater extend of the degree of excess of probabilistic mass that is captured at the tails of the distribution, which is measured by the tail index parameter. This means that the more negative is the tail index the greater is the resulting CVaR because the tail of the returns distributions is heavier. The empirical evidence shows that the characteristic exponent generally takes values between \(1 < \alpha < 2\) for the insurance sector, while for financial applications takes values between \(1.5 < \alpha < 5\).  

4. Empirical results

4.1. Basic Statistical Characteristics and Tail Diagnosis

The Mexican and Brazilian stock markets are no exception to the presence of heavy tails in their time series. In the long-run, during the period under analysis, the stock market indexes from Brazil and Mexico experienced a significant growth, particularly in the case of Brazil. The boom experienced by these two emerging markets was partly induced by liberalization and deregulation policies undertaken by local authorities especially during the last two decades from the XX Century in order to improve their efficiency and liquidity, as well as to attract international portfolio investments (Cabello, de Jesús and Ortiz, 2006).

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4 An important fact that must be pointed out is that the tail index does not have anything relation with the scale parameter of the GEVD.

5 All distributions with heavy tails were defined in this manner by Gnedenko (1943).

6 For more details of CVaR measure based extreme value theory, see Embrechts et al (1998).
The presence of heavy tails can be diagnosed examining the basic statistical characteristics of the return series. To work with stationary series distributed identically and independently, the empirical analysis must be based on continuous returns: 
\[ R_t = \ln(P_t) - \ln(P_{t-1}) \],
where \( R_t \) = return for day \( t \) and \( P_t \) = closing price for day \( t \).

Table 1 summarizes the basic statistical characteristics of the return series from the stock markets under analysis. Both series present positive mean returns and high volatility. Higher return and volatility corresponded to the Brazilian market, 0.361 percent and 3.10 percent, respectively. The Mexican stock market mean return was of 0.131 percent and its standard deviation averaged 1.83 percent.\(^7\) High returns and high volatility in these markets, as reported in this work, can be attributed not only to liberalization and deregulation, but especially to high inflation rates experienced in these countries; nevertheless it is worth pointing out that the stock markets from these countries constituted a hedge against inflation (Cabello, de Jesús and Ortiz 2006). Maximum and minimum returns confirm the volatility of the Brazilian and Mexican exchange markets; Brazil shows a wider differential: 30.79 percent to -25.20 percent; the maximum and minimum returns for the case of Mexico were 23.58 percent and -20.24 percent.

At any rate the statistical characteristics of both markets are very similar. Two facts must be underlined; the lowest return for the Mexican stock Market took place November 16, 1987, almost one month later that the world stock market crash initiated with the fall form the S&P which took place October 19, 1987; in the case of Brazil the minimum return occurred March 23, 1990, when the nation was experiencing inflation rates above 100 percent in annual terms.

Both series also present an asymmetrical behavior. As shown in Table 1: this implies that the tails from the return series present different characteristics. Both markets present positive skewness, 0.27 Brazil and 0.099 Mexico; this means that the right tail of the empirical returns distributions is slightly heavier than the left tail, particularly for the case of the Brazilian stock exchange, i.e. extreme positive returns usually take place more frequently than extreme negative returns. This results from the explosive growth experienced by both markets during the last two decades. In this respect, evidence from negative skewness presented by Duffie and Pan (1997) cannot be confirmed for the stock markets from Brazil and Mexico.

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\(^7\) This analysis is based on nominal returns; inflation adjusted returns cannot be used due to the lack of high frequency (daily) inflation information.
Table 1

**Basic Statistics for Daily Returns from the Stock Markets from Brazil and Mexico**

<table>
<thead>
<tr>
<th>Market</th>
<th>Brazil</th>
<th>Mexico</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mean</td>
<td>0.361</td>
<td>0.131</td>
</tr>
<tr>
<td>Maximum</td>
<td>30.792</td>
<td>23.583</td>
</tr>
<tr>
<td>Minimum</td>
<td>-25.199</td>
<td>-20.242</td>
</tr>
<tr>
<td>Std. Deviation</td>
<td>3.104</td>
<td>1.834</td>
</tr>
<tr>
<td>Skewness</td>
<td>0.265</td>
<td>0.099</td>
</tr>
<tr>
<td>Kurtosis</td>
<td>8.716</td>
<td>25.159</td>
</tr>
<tr>
<td>Jarque-Bera</td>
<td>12005*</td>
<td>177977*</td>
</tr>
<tr>
<td>Probability</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Observations</td>
<td>8745</td>
<td>8698</td>
</tr>
</tbody>
</table>

Note: The Basic statistics for daily returns from the stock markets from Brazil and Mexico are expressed as percent points. An * indicates statistical significance at the 1% confidence level.

Observing the fourth moment estimates, both financial series show an extremely high and statistically significant kurtosis. The highest kurtosis corresponded to the Mexican market, 25.16 points, while the Brazilian stock market had a kurtosis of 8.71 points; at any rate this coefficients mean that tails from the return distributions from both markets are heavier than the normal distribution because they are characterized by a major probabilistic density. In the financial literature the presence of leptokurtosis has been significantly stronger among high frequency returns than for low frequency returns, or else than long period returns. Absence of normality is also confirmed by the Jarque-Bera (1980) statistic which follows a chi-square distribution with two degrees of freedom. The evidence rejecting the assumption of normality for the daily return series is notorious, particularly for the Mexican stock market, for the Jarque-Bera statistics is extremely high (177977) and statistically significant. This result is contrarian to the evidence reported in the financial literature, which demonstrates that the lack of normality at emerging markets is less pronounced when low frequency data is used (Susmel, 2001).

Another procedure to identify the main structure of a financial market and determine if its distribution exhibits heavy tails is the QQ-plot (quantile-quantile plot). Basically the QQ plot compares the quantiles from the empirical distribution with the quantiles from a given

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8 Empirical evidence proving that the returns on shares present excess kurtosis include Fama (1965); Praetz (1972); Blattberg y Gonedes (1974); Kon (1984); Tucker (1992); Kim y Kon (1994). For evidence on exchange rates see: Boothe y Glassman (1987); Tucker y Pond (1988).
distribution, that is, the quantile is the inverse $F^{-1}$ to the distribution function $F$. For instance, if $F$ is continuous then $1\%$–quantile is the number $x$ such that $F(x) = 0.01$. In general,

$$F^{-1}(y) = \inf\{t : F(t) \geq y\}.$$

Figure 1 clearly shows that both the Mexican and Brazilian returns distributions present heavier tails and sharper distribution forms than the normal distribution. If returns are distributed normally, all points from the empirical series should remain on the line depicting quantiles from the normal distribution. However, points from the empirical distribution deviate at the extremes from the straight line, that is the extreme points show more variability than the points captured at the center. The typical S shaped curve shows that the returns distributions from the Mexican and Brazilian exchange markets present a leptokurtic and asymmetrical behavior compared with the normal distribution.

![Figure 1. QQ Plot for Daily Stock Market Returns from Brazil and Mexico](image)

Finally, Figure 2 shows that the two emerging Latin American markets present periods of relative tranquility; stock market prices are relatively stable, but they are followed by volatile periods of varied intensity; changes in the prices are large and generally take place in clusters. This implies that the series of these two markets are time dependent, i.e., present a conditional heteroscedastic behavior, commonly know as the clustering effect, one of the more important characteristics of financial series, demonstrated in the seminal works by Engle (1982) and Bollerslev (1986).
Summing up, returns series from the Mexican and Brazilian stock markets constitute an excellent showcase to analyze the relationship between the behavior of extreme returns and financial risk in extremely volatile markets presenting distributions with heavy tails because these two markets present empirical distributions more leptokurtic than those from mature stock markets from industrialized countries.

Figure 2. Brazilian and Mexican Stock Market Returns, January 2, 1970 – December 31, 2004

4.2 GEVD Parameters for the Stock Markets from Brazil and Mexico

To analyze the behavior of extreme returns for the stock markets from Brazil and Mexico, this paper applies the technique of block maxima. As previously explained this consists in dividing the total series sample in subsamples of a determined number of observations and where the extreme maximum and minimum values are chosen. The number of extreme returns in each subsample depends on the determination of the size of the subsamples and, of course, in the total sample size. Here, extreme daily returns are chosen for different time horizons, one month (n=20), a quarter (n=60) a semester (n=120) and a year (n=252). Once the subsamples have been constructed, the parameters for the distribution of extreme values are estimated, for each of the tails of the return distribution. The parameters of location, scale and form are estimated by the technique of maximum likelihood, \(^9\) independently for each tail of the returns distribution.

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\(^9\) In this case numerical methods and computer algorithms must be used to obtain the maximum estimates because the log-likelihood function becomes a multidimensional non-linear optimization problem.
Table 2 presents the results of the maximum likelihood estimators for the unknown parameters for the GEVD, for maximum and minimum extreme values for the stock markets under analysis, as well as their Standard errors: The evidence shows that the location parameter increases as the size of the bloc sample increases. In the case of the Brazilian index the location parameter presents a change in absolute terms from 3.928 to 8.235 for positive returns and from -3.486 to -8.354 for negative returns. In the case of the Mexican stock exchange the parameter increases from 2.066 to 5.382 for positive returns and from -1.716 to -5.174 for negative returns. This significant increases in the average of extreme positive and negative returns derives from both financial instabilities as well as from the explosive growth experienced by those two emerging markets, which can be attributed to internal and external shocks, liberalization and deregulation, and the hyperinflation rates which characterized Brazil and Mexico in different occasions during the period under analysis, affecting nominal returns. Nonetheless there is a significant difference between the exchange markets from Brazil and Mexico in relation to the value of the location parameter, Considering a block of 120 days, the estimated parameter for positive returns for the Sao Paolo stock market is approximately 1.59 times greater that the Mexican stock market parameter; for negative returns the location parameter for the Brazil is 1.63 times grater that the Mexican parameter.

Estimations for the scale parameter show similar tendencies. An increase in the parameters can be observed as the size of the subsample increases. Considering the right tail of the returns distribution, the scale parameter for the Brazilian stock exchange changes from 2.349 to 3.337; the Mexican stock exchange presents a lower change, from 1.285 y 2.984. The absolute change for the scale parameter for the right tail for both countries is slightly lower, for the case of Brazil the change is from 2.10 to 2.981 and from 1.133 to 2.686 for the Mexican shares market. These results indicate that these emerging markets are subject to high risks, which is a characteristic of economies with fragile financial and economic structures. Comparing further the Brazilian and Mexican stock markets, considering semester subsamples, it can be observed that the scale parameters from Brazil are 1.29 y 1.20 times greater, approximately, than the values obtained for the Mexican markets, for the left and right tails, respectively.

The tail index is the parameter that best explains the historical behavior of the tails of the generalized extreme values distribution of returns of the financial series under analysis. The estimated tail indexes for different sample sizes for positive returns from the Brazilian and
Mexican markets present changes from $-0.121$ to $-0.337$; and $-0.119$ to $-0.278$, and from $-0.235$ to $-0.351$, respectively.

Analyzing Sherman's goodness of fit test, in both markets it can be observed that the hypothesis that maximum and minimum returns follow a generalized extreme values distribution cannot be rejected at a confidence level of five percent, except for monthly returns. These findings confirm the fact that the extreme value distribution explains adequately the asymptotic behavior of maximum and minimum returns. However, even though the tail indexes show increasing negative values, with high standard errors as the subsample size increases. Results from the likelihood ratio test indicate that the tail indexes are significantly different than zero for a confidence level of one percent, for monthly, quarterly, a semester subsamples of the maximum and minimum returns for both emerging markets (with the exception of semester blocks for negative returns for the Brazilian market). This indicates that the asymptotic distribution is at the maximum domain of attraction of the Fréchet distribution (distribution of heavy tails) generally used to model real financial data. Because the emerging Latin American economies experienced dramatic booms and crisis returns of these markets are incompatible with normality. These results are consistent with other empirical studies which have applied classic extreme value theory to returns from other stock markets (Longin, 2000; Ho et al 2000, da Silva y Mendes, 2003).

The Brazilian and Mexican evidence also show that the distributions of daily returns have a light asymmetrical distribution. The right tail of the Brazilian returns is slightly heavier for periods from one month to a semester, but for a one year period the right tail in notoriously more stable and heavier than the left tail, albeit the tail index is not significantly different from zero ($-0.337$ contra $-0.278$). Considering the Mexican stock market tail index it can be observed that the right tail tends to be heavier than the left tail for any subsample period. Thus the Mexican market behaves contrarian to the behavior from stock markets from industrialized countries where the left tail tends to be much heavier than the right tail (Longin, 2000). Similarly estimations for the tail index for positive and negative returns for the Mexican stock market are more stable than the estimates for the Sao Paolo stock market, the former has lower estimates. Two important facts determining this differentiated behavior are the stock market crash that took place November 16, 1987 in Mexico ($-20.24\%$ return) and the March 21, 1990 that took place in Brazil ($-25.20\%$ return). Taking out these observations from the statistics of extreme order, their impact can be detected immediately; the value of the tail indexes change to from $-0.320$ to $-0.18$.
0.267 for Mexico and from −0.221 to −0.159 for Brazil, while the estimators for the localization
and scale parameters change only slightly. The same effect takes place for maximum returns
when the maximum values are omitted; this is an indication that the distribution seemingly has
less heavier tails for one semester periods, another interesting finding concerning the tail
estimators is that the distribution from the Mexican stock exchange are heavier than the returns
from the Brazilian stock market for any time interval, even though returns from the Sao Paolo
market are greater and adjacent; but the distance between these returns is very close.

### Table 2
Maximum Likelihood Estimates for GEVD for Daily Maximum and Minimum Returns for the Stock
Markets from Brazil and Mexico

<table>
<thead>
<tr>
<th>Subsample Size</th>
<th>Location Parameter</th>
<th>Scale Parameter</th>
<th>Tail Index</th>
<th>Sherman Statistic</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Maximum Returns</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>Brazil</strong></td>
<td></td>
<td></td>
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<td></td>
</tr>
<tr>
<td>n = 20 , m = 420</td>
<td>3.928 (0.040)</td>
<td>2.349 (0.031)</td>
<td>-0.121* (0.039)</td>
<td>1.966(2.46%)</td>
</tr>
<tr>
<td>n = 60 , m = 140</td>
<td>5.346 (0.086)</td>
<td>2.898 (0.067)</td>
<td>-0.159* (0.069)</td>
<td>1.203(11.4%)</td>
</tr>
<tr>
<td>n = 120 , m = 70</td>
<td>6.700 (0.131)</td>
<td>3.085 (0.106)</td>
<td>-0.222* (0.108)</td>
<td>-0.350(36.3%)</td>
</tr>
<tr>
<td>n = 252 , m = 35</td>
<td>8.235 (0.243)</td>
<td>3.378 (0.154)</td>
<td>-0.337 (0.174)</td>
<td>-0.350(36.3%)</td>
</tr>
<tr>
<td><strong>Mexico</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>n = 20 , m = 420</td>
<td>2.066 (0.033)</td>
<td>1.285 (0.027)</td>
<td>-0.284* (0.043)</td>
<td>1.984(2.36%)</td>
</tr>
<tr>
<td>n = 60 , m = 140</td>
<td>3.130 (0.054)</td>
<td>1.804 (0.045)</td>
<td>-0.313* (0.074)</td>
<td>1.158(12.4%)</td>
</tr>
<tr>
<td>n = 120 , m = 70</td>
<td>4.207 (0.102)</td>
<td>2.386 (0.088)</td>
<td>-0.365* (0.122)</td>
<td>-0.152(56.0%)</td>
</tr>
<tr>
<td>n = 252 , m = 35</td>
<td>5.382 (0.233)</td>
<td>2.948 (0.129)</td>
<td>-0.388 (0.174)</td>
<td>-1.119(86.9%)</td>
</tr>
<tr>
<td><strong>Minimum Returns</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>Brazil</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>n = 20 , m = 420</td>
<td>-3.486 (0.036)</td>
<td>2.100 (0.028)</td>
<td>-0.119* (0.040)</td>
<td>1.982(2.37%)</td>
</tr>
<tr>
<td>n = 60 , m = 140</td>
<td>-5.349 (0.074)</td>
<td>2.479 (0.057)</td>
<td>-0.134* (0.068)</td>
<td>1.320(9.33%)</td>
</tr>
<tr>
<td>n = 120 , m = 70</td>
<td>-6.642 (0.111)</td>
<td>2.505 (0.073)</td>
<td>-0.197 (0.090)</td>
<td>-0.296(38.3%)</td>
</tr>
<tr>
<td>n = 252 , m = 35</td>
<td>-8.354 (0.185)</td>
<td>2.981 (0.135)</td>
<td>-0.278 (0.152)</td>
<td>-1.504(93.4%)</td>
</tr>
<tr>
<td><strong>Mexico</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>n = 20 , m = 420</td>
<td>-1.712 (0.029)</td>
<td>1.138 (0.023)</td>
<td>-0.235* (0.043)</td>
<td>1.826(3.39%)</td>
</tr>
<tr>
<td>n = 60 , m = 140</td>
<td>-2.835 (0.046)</td>
<td>1.531 (0.037)</td>
<td>-0.251* (0.073)</td>
<td>1.477(6.98%)</td>
</tr>
<tr>
<td>n = 120 , m = 70</td>
<td>-4.099 (0.087)</td>
<td>2.096 (0.067)</td>
<td>-0.320* (0.096)</td>
<td>-0.336(36.8%)</td>
</tr>
<tr>
<td>n = 252 , m = 35</td>
<td>-5.174 (0.179)</td>
<td>2.686 (0.128)</td>
<td>-0.351 (0.176)</td>
<td>-0.803(78.9%)</td>
</tr>
</tbody>
</table>

Values in parenthesis represent standard errors for the estimates of maximum likelihood for the
GEVD, derived from the diagonal of the variance-covariance matrix and calculated with a
subroutine in C language. The last column shows results from the test of goodness of fit
developed by Sherman; the p value is within parenthesis. The tail index estimates marked with
an asterisk are significantly different from zero at a confidence level of one percent.
Another relevant result that must be pointed out, derived from the tail index estimators, is that there exist second moments for the superior and inferior tails of the returns distributions for both stock market indexes; in other words these distributions have finite variances, but with different characteristics for each one of the tails since the estimated value for the tail indexes for the extreme positive and negative returns is greater than \(-0.50\) for any time interval; this indicates that the log-likelihood function is regular and that the estimators of maximum likelihood all have the usual asymptotic properties demonstrated by Smith (1985). The fact the returns distribution present heavy tails does not mean that higher moments do not exist. For instance the first four moments exist for the returns distribution of the Brazilian stock market when extreme returns are chosen for subsamples of less than one year because the estimated tail index is above \(-0.25\). In the case of the Mexican stock market index the first four moments only exist for extreme negative returns for one month periods. Similarly the first three moments exist for extreme positive returns for monthly periods, too.

4.3 VaR and CVaR Evidence

Research works dealing with risk measuring have mainly focused on the negative returns captured at the left tail of probability distribution. However, empirical analysis based on EVT yields information from both tails of the distribution. Thus, VaR and CVaR can be determined for long and short positions. In the first case an investor could take losses when prices decrease due to liquidity needs; in the second case an investor can experience losses when the assets prices increase. This generally takes place when investors sell short. In this context VaR and CVaR estimations for a long position concentrate on the left tail, and on the right tail for a short position, where positive and negative returns are captured, respectively.

Finally VaR and CVaR under EVT are generally estimated for confidence levels of 95%, 97.5%, 99% and 99.9%. Results are then compared with results from conventional first generation parametric and non parametric models. Parametric models include the delta-normal model with unconditional normal distribution, GARCH(1,1) and the method of weighted moving averages with conditional normal distribution. Non-parametric methods include the method of historical simulation and Monte Carlo simulation. In this work only the historical simulation method is used. The Monte Carlo methodology is not used to measure market risk because the risk factors considered are lineal.
Results for the five VaR and CVaR models that include the tails of the returns distributions for the stock markets from Brazil and Mexico are included in Table 3. In both markets can be observed that the parametric estimates based on conditional and unconditional returns distributions with normal innovations, yield more conservative VaR results for both the long and short positions, that is, greater losses than the parametric estimates based on an unconditional extreme value distribution, for smaller confidence levels. For instance at a confidence level of 95% the maximum delta normal VaR loss on a $US100.00 long position\textsuperscript{10} is US$2.39; Applying the conditional GARCH and weighted moving averages models losses equal US$1.93 and US$1.89 respectively. However the value at risk estimated using EVT equals US$1.21 only. In the case of a long position at the Sao Paolo market estimate losses according to the conditional and unconditional return distributions equal $3.47, $2.44 and $2.41; using the asymptotic extreme values distribution the loss equals $2.95. Thus, estimates using VaR based on EVT are more conservative.

For the short position similar results were obtained for both emerging markets. On the contrary for tighter confidence levels the value at risk estimated based on EVT are greater than the conventional models. For a confidence level of 99.5% the VaR estimates for both markets based on EVT for a short position are approximately two times greater than the VaR estimates obtained with the delta normal model; $11.85 vs. $4.43 for the Mexican stock market and $15.06 vs $7.21 for the Brazilian stock market. For a long position, VaR estimates derived from the alternative models under consideration are similar to the short position contrasts previously examined: $10.46 vs $4.49 (Mexico) and $13.47 vs $6.51 (Brazil). This difference can be explained by the way the distribution of extreme returns is derived because the estimated tail index is always negative for both markets. That is, the evidence clearly justifies the advantage of estimating VaR based on a generalized extreme value distribution, for high confidence levels.

A comparative analysis between the historical simulation methodology and the extreme value estimates clearly show that models based on empirical distributions yield more conservative VaR estimates for any confidence level and any financial position in the market, that is, risk is overestimated.\textsuperscript{11} This is clearly reflected in the Sao Paolo market than in the Mexican market, for positive returns: $22.73 vs. $15.06 for Brazil, and $19.08 vs $11.85 for

\textsuperscript{10} Assuming a U.S. investor takes long and short positions at several emerging markets.

\textsuperscript{11} When the product between the confidence level and the simple size ($cT$) is not a whole number the technique of linear interpolation is used to obtain more precise VaR estimates or the maximum loss by the historical simulation method.
Mexico. The main reason for this fact is that the empirical distribution is usually heavier at its interior because extreme returns are clearly discrete. This implies that VaR estimates that depend on tails are calculated in a discrete way, with a high variance, inducing overestimations or underestimations by the non-parametric method. The historical method cannot yield good estimates beyond the sample where estimates based on EVT guarantee more precise risk estimates.

Examining the empirical CVaR results reported in Table 3 it can be observed that there also difference for the estimates for each country, particularly for the simulation and extreme value models. CVaR based on extreme values for a $100.00 position and a confidence level of 99% equals $5.47 for a long position and $5.95 for a short position at the Mexican market; in the case of Brazil the results are $8.01 for the long position and $7.89 for the short position. The historical VaR estimates are $10.09 and $10.63 (Mexico) and $11.65 and $14.60 (Brazil).
Based on a normal inconditional distribution the estimates are $3.88$ y $3.82$ (Mexico) and $5.62$ and $6.22$ (Brazil). The conditional methods yield smaller and identical CVaR estimates. For a confidence level of 99.5% show that CVaR are more conservative, implying that potential losses are greater, particularly for positive returns for the case of both markets, due to their explosive growth. This means that risk at emerging markets, because their instabilities and severe and persistent, present an opposed behavior than the one characteristic of mature markets.

This empirical characteristic can be better appreciated considering the difference between VaR and CVaR. For instance, applying the extreme value approximation for confidence levels of 99.0% and 99.9% it can be observed that the difference increases, in absolute terms from $1.75$ to $4.92$ for the long position and from $2.18$ to $6.82$ for the short position in the case of Mexico; and from $1.78$ to $3.84$ for the long position and from $1.75$ to $4.30$ for the short position in the case of Brazil. From a statistical point of view, this difference in absolute terms can be attributed to the tail index value because the more negative it is the greater is its impact in the value of conditional risk. This effect is also present for the application of the historical model, except for the Mexican case where the difference diminished in absolute terms from $3.82$ to $1.10$ and from $4.07$ to $3.92$ for the long and short positions, respectively. Causing this significant increase is the fact that all observations that exceed the value at risk are considered. In relation to the delta normal model, the difference also decreases from $0.50$ to $0.41$ and from $0.49$ to $0.40$ for the long and short positions at the Mexican market; and from $0.72$ to $0.59$ and from $0.80$ to $0.66$ for the long and short positions at the Brazilian market.

The above results clearly show that CVaR estimates based on EVT and the historical simulation tend to deviate from VaR as the confidence level becomes more restrictive, while the CVaR estimated on a normal distribution tends to converge to the VaR estimate. This is, any model assuming normality does not only yield incorrect information about tail risks of the returns distributions, but also underestimates the value of conditional risk. Hence, the application of EVT is very important to correctly model the tails from the distributions of returns at financial markets.\textsuperscript{12}

Finally, VaR and CVaR estimated based on extreme values clearly show that the Sao Paolo stock market was riskier during the period under analysis, for both the short and long

\textsuperscript{12} This analysis was also carried out for low frequency data for a ten days investment horizon. Results are even more conservative. They can be obtained contacting the authors.
positions and for any confidence level. This finding is not fully supported by the value of the tail index, since both the left and right tails of the IPC distribution returns seemingly are heavier than the tails from the BOVESPA returns distribution. For high frequency (daily) data, tail indexes for the Mexican stock exchange are smaller or negative when compared with the tail indexes from the Sao Paolo stock market: –0.320 vs. –0.221 for daily positive returns and; –0.365 vs. –0.222 for daily negative returns. This situation can be also observed comparing the left and right tails of the returns distributions for both Latin American markets, particularly for the case of Brazil market. For confidence levels of 95% y 97.5%, potential losses for the long position are greater than potential losses for the short position; for example $2.95 vs $2.09 and $4.23 vs. $3.66, for the stated confidence levels, even though the estimated tail indexes are very similar. Considering the case of Mexico’s potential losses are $1.21 vs. $1.04 and $2.14 vs. $2.03, for a 120 days subsample. These results imply that heavier tails do not necessarily mean higher risk. Thus, VaR and CVaR estimates seemingly are not exempt from tail risk when the asymptotic behavior of stock market indexes is described by a distribution of extreme values or heavy tails.

Conclusions

This paper has shown that EVT has several advantages to quantify in a more precise manner potential losses or tail risk form the distribution for the emerging stock markets from Brazil and Mexico, for high confidence levels. Particularly, for the stock market from Sao Paolo where the evidence found that market fluctuations are large and adjacent due to an explosive growth, coupled with hyper inflation rates.

Main findings of this study indicate first that both stock markets are characterized by heavy tails due to excess kurtosis this can be explained by the recurrent crisis that the Mexican and Brazilian economies experienced during the last decades. This fact is also supported by the negative tail index values of the generalized extreme value distribution demonstrating that the asymptotic distribution for extreme values is at the maximum domain of attraction of the Fréchet distribution, also known as a distribution of heavy tails, which is generally used to model real financial returns. Also, the right and left tails present different characteristics due to existing bias or asymmetry; for this reason the level of risk shows a different behavior than mature markets because at emerging market atypical movements are more frequent as a result of sharp changes in exchange rates in the long and short run.

13 Examples about the impact of tail risk on VaR and CVaR estimates can be found in Yamai y Yoshima (2002a, 2002d).
Second, the empirical evidence clearly shows that VaR and CVaR estimates based on EVT yield more precise and robust information about financial risk than conventional parametric approximations, for confidence levels of 99% and 99.9%. This derives from the fact that the GEVD correctly models the magnitude of extreme movements found both at the right and left tails of the probability distribution. It is worth mentioning that values based on empirical distributions yield more conservative risk estimates that those base of heavy tail distributions; however they are affected by a high variance derived from the discrete behavior of returns found outside the tails. Further, estimates based on EVT give better information about risk outside the sample for more conservative confidence more levels (99%). Also, VaR and CVaR estimates based on EVT are more conservative for short and long positions taken at the Sao Paolo bourse than those from the Mexican stock market; however, are not statistically supported, since both tails of the Mexican market are heavier than the tails from the Sao Paolo market, which results in model risk, particularly for the VaR estimate, since the CVaR estimate is characterized by better proprieties concerning tail risk.

Thus, VaR and CVaR estimates based on EVT can be used effectively to manage financial risk in an univariate context; they allow investors attain a better perspective about risks assumed due to extreme movements and they take buy and sell decisions under uncertainty in both mature and emerging markets.

Summing up, VaR and CVaR based on EVT can become very valuable tools to manage portfolio holdings risk. Furthermore, risk analysis base on EVT also has valuable uses for the banking industry and its regulatory authorities to determine more efficiently capital needs. Like any other model, EVT has some shortcomings. An important weakness refers to the estimation of extreme values by block maxima method; that might lead to subestimate risk during financial turbulence periods because some important extreme returns might be left out of the sample. Finally another limitation is that risk is estimated in an individual and static way; does not consider the stochastic volatility present in most financial series. This is a matter that must be dealt in future financial research.
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